

# CAN BARYOGENESIS SURVIVE IN THE STANDARD MODEL DUE TO STRONG HYPERMAGNETIC FIELD?

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## ABSTRACT

The electroweak phase transition in a constant hypermagnetic field is studied in the Standard Model. The symmetry behaviour is investigated within the consistent effective potential of the scalar and magnetic fields at finite temperature. It includes the one-loop and ring diagram contributions. All fundamental fermions and bosons are taken into consideration with their actual masses. The only free parameter is the Higgs boson mass which is chosen to be in the energy interval  $75 \text{ GeV} \leq m_H \leq 115 \text{ GeV}$ . It is found that for the field strengths  $H \sim 10^{22} - 10^{23} \text{ G}$  the electroweak phase transition is of first order but a baryogenesis condition is not satisfied. For stronger fields it becomes of second order. Hence it is concluded that the smooth hypermagnetic field does not generate the strong first order phase transition and the baryogenesis does not survive in the Standard Model. The comparison with the results of other approaches is done.

## 1 Introduction

Among nowadays problems of high energy physics there are two ones which, at first glance, are not connected with each other. These are the value of the Higgs boson mass  $m_H$  and the magnetic field strengths  $H$  which can be present in the early universe (see surveys [1], [2]). They are of paramount importance for particle physics and cosmology. For instance, a large scale homogeneous hypercharge magnetic field could essentially influence the type of the electroweak (EW) phase transition making it strong first order [4], [5]. An interest to this problem increased recently when it has been realized that without external fields a standard baryogenesis does not hold in the minimal Standard Model (SM).

In Refs. [4], [5], [6], [7] the influence of the constant hypermagnetic field on the EW phase transition has been investigated. In the former one the EP was computed in a tree approximation and the result that the presence of  $H_Y$  makes the weak first-order phase transition stronger has been derived. In Ref. [5] the temperature dependent part of the EP was calculated in one-loop order whereas

the external field has also been allowed for in tree approximation. Therein it was found that for the field strengths  $H_Y > 0.3 - 0.5T^2$ , where  $T$  is temperature at the transition, the standard baryogenesis survives. As this paper is concerned, we would like to notice that for the weak first-order phase transition the fluctuations are essential, the one-loop approximation to the EP is not trusty and the correlation corrections must be included [9], [10], [11]. Moreover, no investigation of the EP curve in strong fields has been carried out in Ref. [5]. In fact, it was just assumed that the phase transition is of first order and the jump of the order parameter is as at the zero external field. Furthermore, the role of fermions at high temperature and strong fields has not been investigated. The same has relevance to the papers [4], [6], [7]. But as it will be seen in what follows, fermions significantly influence a vacuum dynamics in the environment.

To make a link between recent studying of symmetry behaviour in the external hypermagnetic field and the already obtained results for the case of usual magnetic field [12], [15], we notice that in the broken phase  $H_Y$  is connected with  $H$  by the relation  $H = H_Y \cos \theta$ , where  $\theta$  is the Weinberg angle. So, all investigations dealing with symmetry behaviour in the magnetic fields at high temperature are relevant to the considered case of  $H_Y$  in the respect of the form of the EP curve at different  $T, H_Y$ . The hypercharge field influences the scalar field condensate at tree level, as magnetic field in the Higgs model, and acts to restore symmetry. That was the reason why it has been allowed for in lower order in Ref. [4]. But, as we will see for strong fields and heavy  $m_H$ , the form of the EP curve in the broken phase is very sensitive to the change of the parameters. In the restored phase, there is a number of terms having order  $\sim (gH_Y)^{3/2}T$  which can influence the temperature of the phase transition. So, to have an adequate picture of the phenomenon investigated the radiation corrections in the field must be calculated in both the broken and the restored phases.

In the present paper the EW phase transition in the constant strong hypercharge magnetic field is investigated within the consistent EP including the one-loop and ring diagram contributions. All bosons and fermions are taken into account with their actual masses (in particular, the t-quark mass is 175 GeV). So, the only free parameter remains the mass  $m_H$ . We assume it to be in the energy range  $75 \text{ GeV} \leq m_H \leq 115 \text{ GeV}$ , in order to take account of the modern experimental low limit  $m_H \geq 90 \text{ GeV}$ . We calculate the contribution of ring diagrams in the external fields. As it is well known, these diagrams cancel the imaginary terms of the one-loop effective potential and the total potential is real at sufficiently high temperatures [27]. Due to the ring diagrams in the field, there is also cancellation of an instability generated in strong magnetic fields in the  $W$ -boson sector. This instability appears because of the presence in the  $W$ -boson spectrum of the tachyonic mode  $\epsilon^2 = p_3^2 + M_w^2 - eH$  (see survey [35]). This mode is the transversal one. So, to treat the problem carefully the ring diagrams of the mode with the transversal effective mass at nonzero  $H, T$  have to be added. This requires calculation of the  $W$ -boson mass operator at high temperature and

strong fields. We present the relevant results also. With such the term included the total EP is real at sufficiently high temperatures and suitable to investigate symmetry behaviour.

For the bosonic part of the Salam-Weinberg model the phase transitions in magnetic fields at high temperature have been studied in one-loop approximation in Refs. [20]- [23], [25], [26], [16]. However, the aspect of the EW phase transition, which was not investigated, is the influence of the correlation corrections described by the ring diagrams at high temperatures and strong fields. At zero field it was studied in detail in Refs. [27], [9], [10] but for not heavy mass  $m_t$ . In particular, in Ref. [9]  $m_t$  was chosen of order  $\sim 110$  GeV. So, for the present day experimental data it should be revised.

In what follows, considering EP in the broken phase we will write  $H$  for the usual external magnetic field remembering that it equals to  $H = H_Y \cos\theta$ . The  $Z$ -component of the field  $H_Y$  is screened by the scalar field condensate  $\phi_c$ . The constant external field is a good approximation for the description of the initial stage of the first order EW phase transition when the bubbles are not large, as it was discussed in Ref. [5].

The content is as follows. In Sects. 2, 3 the one-loop contributions of bosons and fermions to the EP  $V^{(1)}(T, H, \phi_c)$  are calculated in the form convenient for numerical investigations. In Sect. 4 we compute the contributions of ring diagrams. In Sect. 5 the EP for the restored phase is calculated. In Sect. 6 symmetry behaviour at high temperatures and strong external fields is investigated and it is shown the EW phase transition is of first order for the field strengths  $H \sim 10^{22} - 10^{23}$  G but the baryogenesis condition is not satisfied. For stronger fields it becomes of second order. Thus, we come to conclusion that in the SM baryogenesis does not survive under smooth external hypermagnetic field. The comparison of our results with that of other approaches and discussion are given in Sects. 6, 7.

## 2 Boson contributions to $V^{(1)}(T, H, \phi_c)$

The Lagrangian of the boson sector of the Salam-Weinberg model is

$$L = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + (D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{m^2}{2}(\Phi^\dagger\Phi) - \frac{\lambda}{4}(\Phi^\dagger\Phi)^2, \quad (1)$$

where the standard notations are introduced

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\varepsilon^{abc}A_\mu^b A_\nu^c, \\ G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\ D_\mu &= \partial_\mu + \frac{1}{2}igA_\mu^a\tau^a + \frac{1}{2}ig'B_\mu. \end{aligned} \quad (2)$$

The vacuum expectation value of the field  $\Phi$  is

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ \phi_c \end{pmatrix}. \quad (3)$$

The fields corresponding to the  $W$ -,  $Z$ -bosons and photons, respectively, are

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}}(A_\mu^1 \pm iA_\mu^2), \\ Z_\mu &= \frac{1}{\sqrt{g^2 + g'^2}}(gA_\mu^3 - g'B_\mu), \\ A_\mu &= \frac{1}{\sqrt{g^2 + g'^2}}(g'A_\mu^3 + gB_\mu). \end{aligned} \quad (4)$$

To incorporate an interaction with an external hypermagnetic field we add the term  $\frac{1}{2}\vec{H}\vec{H}_Y$  to the Lagrangian. The value of the macroscopic magnetic field generated inside the system will be determined by minimization of free energy. Interaction with a classical electromagnetic field is introduced as usually by splitting the potential in two parts:  $A_\mu = \bar{A}_\mu + A_\mu^R$ , where  $A^R$  describes a radiation field and  $\bar{A} = (0, 0, Hx^1, 0)$  corresponds to the constant magnetic field directed along the third axis. We make use of the gauge-fixing conditions [35]

$$\partial_\mu W^{\pm\mu} \pm ie\bar{A}_\mu W^{\pm\mu} \mp i\frac{g\phi_c}{2\xi}\phi^\pm = C^\pm(x), \quad (5)$$

$$\partial_\mu Z^\mu - \frac{i}{\xi'}(g^2 + g'^2)^{1/2}\phi_c\phi_z = C_z, \quad (6)$$

where  $e = g\sin\theta$ ,  $\tan\theta = g'/g$ ,  $\phi^\pm, \phi_z$  are the Goldstone fields,  $\xi, \xi'$  are the gauge fixing parameters,  $C^\pm, C_z$  are arbitrary functions and  $\phi_c$  is the value of the scalar condensate. In what follows, all calculations will be done in the general relativistic renormalizable gauge (5), (6) and after that we set  $\xi, \xi' = 0$  choosing the unitary gauge.

To compute the EP  $V^{(1)}$  in the background magnetic field let us introduce the proper time (s-representation) for the Green functions

$$G^{ab} = -i \int_0^\infty ds \exp(-isG^{-1ab}) \quad (7)$$

and use the method of Ref. [28], allowing in a natural way to incorporate the temperature into this formalism. A basic formula of Ref. [28] connecting the Matsubara-Green functions with the Green functions at zero temperature is needed,

$$G_k^{ab}(x, x'; T) = \sum_{-\infty}^{+\infty} (-1)^{(n+[x])\sigma_k} G_k^{ab}(x - [x]\beta u, x' - n\beta u), \quad (8)$$

where  $G_k^{ab}$  is the corresponding function at  $T = 0, \beta = 1/T, u = (0, 0, 0, 1)$ , the symbol  $[x]$  means the integer part of  $x_4/\beta$ ,  $\sigma_k = 1$  in the case of physical fermions and  $\sigma_k = 0$  for the boson and the ghost fields. The Green functions in the right-hand side of formula (8) are the matrix elements of the operators  $G_k$  computed in the states  $|x', a\rangle$  at  $T = 0$ , and in the left-hand side the operators are averaged in the states with  $T \neq 0$ . The corresponding functional spaces  $U^0$  and  $U^T$  are different but in the limit of  $T \rightarrow 0$   $U^T$  transforms into  $U^0$ .

The one-loop contribution into EP is given by the expression

$$V^{(1)} = -\frac{1}{2} \text{Tr} \log G^{ab}, \quad (9)$$

where  $G^{ab}$  stands for the propagators of all the quantum fields  $W^\pm, \phi^\pm, \dots$  in the background magnetic field  $H$ . In the s-representation the calculation of the trace can be done in accordance with the formula [24]

$$\text{Tr} \log G^{ab} = - \int_0^\infty \frac{ds}{s} \text{tr} \exp(-isG_{ab}^{-1}). \quad (10)$$

Details of calculations based on the s-representation and the formula (8) can be found, for instance, in Refs. [28], [12], [38]. The terms with  $n = 0$  in Eqs. (8), (9) give the zero temperature expressions for the Green functions and the effective potential  $V^{(1)}$ , respectively. They are the only terms possessing divergences. To eliminate them and uniquely fix the potential we make use the following renormalization conditions at  $H, T = 0$  [12]:

$$\frac{\partial^2 V(\phi, H)}{\partial H^2} \Big|_{H=0, \phi=\delta(0)} = \frac{1}{2}, \quad (11)$$

$$\frac{\partial V(\phi, H)}{\partial \phi} \Big|_{H=0, \phi=\delta(0)} = 0, \quad (12)$$

$$\frac{\partial^2 V(\phi, H)}{\partial \phi^2} \Big|_{H=0, \phi=\delta(0)} = m^2, \quad (13)$$

where  $V(\phi, H) = V^{(0)} + V^{(1)} + \dots$  is the expansion with respect to the number of loops and  $\delta(0)$  is the vacuum value of the scalar field determined in tree approximation.

It is convenient for what follows to introduce dimensionless quantities:  $h = H/H_0$  ( $H_0 = M_w^2/e$ ),  $\phi = \phi_c/\delta(0)$ ,  $K = m_H^2/M_w^2$ ,  $B = \beta M_w$ ,  $\tau = 1/B = T/M_w$ ,  $\mathcal{V} = V/H_0^2$  and  $M_w = \frac{g}{2}\delta(0)$ .

After the reparametrization the scalar field potential is explicitly expressed in terms of the ratio  $K$ ,

$$\mathcal{V}^{(0)} = \frac{h^2}{2} + K \sin^2 \theta (-\phi^2 + \frac{\phi^4}{2}). \quad (14)$$

Remind that  $h$  is the electromagnetic component of the hypercharge field  $h_Y$  which is unscreened in the broken phase. In the restored phase it will be convenient to work in terms of the initial fields and we will carry out the corresponding calculations later.

The renormalized one-loop EP is given by the sum of the functions

$$\mathcal{V}_1 = \mathcal{V}^{(0)} + \mathcal{V}^{(1)}(\phi, h, K) + \omega^{(1)}(\phi, h, K, \tau), \quad (15)$$

where  $\mathcal{V}^{(1)}$  is the one-loop EP at  $T = 0$ , which has been studied already in Ref. [35]. It has the form:

$$\mathcal{V}^{(1)} = \mathcal{V}_{w,z}^{(1)} + \mathcal{V}_\phi^{(1)}, \quad (16)$$

where

$$\begin{aligned} \mathcal{V}_{w,z}^{(1)} = & \frac{3\alpha}{\pi} \left[ h^2 \log \Gamma_1 \left( \frac{1}{2} + \frac{\phi^2}{2h} \right) + h^2 \zeta'(-1) + \frac{1}{16} \phi^4 - \frac{1}{8} \phi^4 \log \frac{\phi^2}{2h} + \frac{1}{24} h^2 \right. \\ & - \frac{1}{24} h^2 \log(2h) \left. \right] + \frac{\alpha}{2\pi} [-2h^2 + (h^2 + h\phi^2) \log(h + \phi^2) \\ & + (h^2 - h\phi^2) \log |h - \phi^2|] + i \frac{1}{2} \alpha h (\phi^2 - h) \theta(h - \phi^2), \end{aligned} \quad (17)$$

$$\begin{aligned} \mathcal{V}_\phi^{(1)} = & K \sin^2 \theta_w \left( -\phi^2 + \frac{1}{2} \phi^4 \right) \\ & + \frac{3\alpha}{4\pi} \left( 1 + \frac{1}{2 \cos^4 \theta} \right) \left( \frac{1}{2} \phi^4 \log \phi^2 - \frac{3}{4} \phi^4 + \phi^2 \right) \\ & + \frac{\alpha K^2}{32\pi} \left[ \left( \frac{9}{2} \phi^4 - 3\phi^2 + \frac{1}{2} \right) \log \left| \frac{3\phi^2 - 1}{2} \right| - \frac{27}{4} \phi^4 + \frac{21}{2} \phi^2 \right] \end{aligned} \quad (18)$$

and  $\omega^{(1)}$  is the temperature dependent contribution to the EP determined by the corresponding terms of formulae (8), (9) with  $n \neq 0$ .

We outline the used calculation procedure considering the  $W$ -boson contribution as an example [38],

$$\omega_w^{(1)} = \frac{\alpha}{2\pi} \int_0^\infty \frac{ds}{s^2} e^{-is(\phi^2/h)} \left[ \frac{1 + 2 \cos 2s}{\sin s} \right] \sum_1^\infty \exp(ihB^2 n^2 / 4s). \quad (19)$$

As Eq. (17), this expression contains an imaginary part for  $h > \phi^2$  appearing due to the tachyonic mode  $\varepsilon^2 = p_3^2 + M_w^2 - eH$  in the  $W$ -boson spectrum [35], [37], [36]. It can be explicitly calculated by means of an analytic continuation taking into account the shift  $s \rightarrow s - i0$  in the  $s$ -plane. Fixing  $\phi^2/h > 1$  one can rotate clockwise the integration contour in the  $s$ -plane and direct it along the negative imaginary axis. Then, using the identity

$$\frac{1}{\sinh s} = 2 \sum_{p=0}^\infty e^{-s(2p+1)}, \quad (20)$$

and integrating over  $s$  in accordance with the standard formula

$$\int_0^\infty ds s^{n-1} \exp(-\frac{b}{s} - as) = 2(\frac{b}{a})^{n/2} K_n(2\sqrt{ab}), \quad (21)$$

( $a, b > 0$ ), one can represent the expression (19) in the form

$$Re\omega_w^{(1)} = -4\frac{\alpha}{\pi}\frac{h}{B}(3\omega_0 + \omega_1 - \omega_2), \quad (22)$$

where

$$\omega_0 = \sum_{p=0}^\infty \sum_{n=1}^\infty \frac{x_p}{n} K_1(nBx_p); \quad x_p = (\phi^2 + h + 2ph)^{1/2}, \quad (23)$$

$$\omega_1 = \sum_{n=1}^\infty \frac{y}{n} K_1(nBy), \quad y = (\phi^2 - h)^{1/2}. \quad (24)$$

We have in the range of parameters  $\phi^2 < h$  after analytic continuation

$$\omega_1 = -\frac{\pi}{2} \sum_{n=1}^\infty \frac{|y|}{n} Y_1(nB | y |), \quad (25)$$

$$\omega_2 = \sum_{n=1}^\infty \frac{z}{n} K_1(nBz), \quad z = (\phi^2 + h)^{1/2}, \quad (26)$$

$K_n(x), Y_n(x)$  are the Bessel functions. The imaginary part of  $\omega_w^{(1)}$  is given by the expression

$$Im\omega_1 = -2\alpha\frac{h}{B} \sum_{n=1}^\infty \frac{|y|}{n} J_1(nB | y |), \quad (27)$$

$J_1(x)$  is the Bessel function. As it is well known, the imaginary part of EP is signaling the instability of the system. In what follows we shall consider mainly symmetry behaviour described by the real part of the EP. As the imaginary part, it will be cancelled in the consistent calculation including the one-loop and ring diagram contributions to the EP.

Carrying out similar calculations for the  $Z$ - and Higgs bosons, we obtain [12]:

$$\omega_z = -6\frac{\alpha}{\pi} \sum_{n=1}^\infty \frac{\phi^2}{\cos^2 \theta_w n^2 B^2} K_2(\frac{nB\phi}{\cos \theta}), \quad (28)$$

$$Re\omega_\phi = \left\{ \begin{array}{l} -2\frac{\alpha}{\pi} \sum \frac{t^2}{B^2 n^2} K_2(nBt) \\ \alpha \sum_{n=1}^\infty \frac{|t|^2}{n^2 B^2} Y_2(nB | t |) \end{array} \right\}, \quad (29)$$

where the variable  $t = [K_w(\frac{3\phi^2-1}{2})]^{1/2}$  at  $3\phi^2 > 1$  and series with the function  $Y_2(x)$  has to be calculated at  $3\phi^2 < 1$ . The corresponding imaginary term is also cancelled as it will be shown below.

The above expressions (16), (22), (28), (29) will be used in the numerical studying of symmetry behaviour at different  $H, T$ . There is a cancellation of a number of terms from the zero-temperature contributions given Eqs. (16) and  $T$ -dependent ones. This fact has a general character and was used in checking of the correctness of calculations.

### 3 Fermion contributions to $V^{(1)}(H, T, \phi_c)$

To find the convenient form of the fermion contribution to the EP let us consider the standard unrenormalized expression written in the  $s$ -representation [33]:

$$V_f^{(1)} = \frac{1}{8\pi^2} \sum_{n=-\infty}^{\infty} (-1)^n \int_0^{+\infty} \frac{ds}{s^3} e^{-(m_f^2 s + \beta^2 n^2 / 4s)} (eHs) \coth(eHs), \quad (30)$$

$m_f$  is a fermion mass. Here, we have incorporated the equation (8) to introduce the temperature dependence. In what follows, we shall take into account the contributions of all fermions - leptons and quarks - with the values of masses equalled to the present day data. Usually, considering symmetry behaviour without external fields one restricts themselves by the  $t$ -quark contribution, only. But in the case of the external field applied this is not a good idea, since the dependence of  $V^{(1)}$  on  $H$  is the complicate function of the parameters  $m_f^2$ ,  $eH$ ,  $T$ . At some fixed values of  $H, T$  fermions with the definite corresponding masses are dominant. For instance, at high temperature the leading term of  $V_f^{(1)}$  is  $\sim H^2 \log \frac{T}{m_f}$ . Hence it follows that light fermions are important. In general, a very complicate dependence on the field takes place. We include this in the total, carrying out numerical calculations and summing up over all the fermions. Now, separating the zero temperature terms by means of the relation  $\sum_{-\infty}^{+\infty} = 1 + 2 \sum_1^{\infty}$  and introducing the parameter  $K_f = m_f^2 / M_w^2 = G_{Yukawa}^2 / g^2$ , we obtain for the zero temperature fermion contribution to the dimensionless EP,

$$\begin{aligned} \mathcal{V}_f(h, \phi) &= \frac{\alpha}{4\pi} \sum_f K_f^2 (-2\phi^2 + \frac{3}{2}\phi^4 - \phi^4 \log \phi^2) \\ &- \frac{\alpha}{\pi} \sum_f (q_f^2 \frac{h^2}{6} \log \frac{2 |q_f| h}{K_f}) \\ &- \frac{\alpha}{\pi} \sum_f [2q_f^2 h^2 \log \Gamma_1(\frac{K_f \phi^2}{2 |q_f| h}) + (2\zeta'(-1) - \frac{1}{6}) q_f^2 h^2 \\ &+ \frac{1}{8} K_f^2 \phi^4 + (\frac{1}{4} K_f^2 \phi^4 - \frac{1}{2} K_f |q_f| h \phi^2) \log \frac{2 |q_f| h}{K_f \phi^2}], \end{aligned} \quad (31)$$



where  $q_f$  is a fermion electric charge, the sum  $\sum_f = \sum_{f=1}^3 (\text{leptons}) + 3 \sum_{f=1}^3 (\text{quarks})$  counts the contributions of leptons and quarks with their electric charges. The function  $\Gamma_1$  is defined as follows [31] (see also survey [35]):

$$\log \Gamma_1(x) = \int_0^x dy \log \Gamma(y) + \frac{1}{2}x(x-1) - \frac{1}{2}x \log(2\pi). \quad (32)$$

The finite temperature part can be calculated in a way described in the previous section. In the dimensionless variables it looks as follows:

$$\begin{aligned} \omega_f = & 4 \frac{\alpha}{\pi} \sum_f \left\{ \sum_{p=0}^{\infty} \sum_{n=1}^{\infty} (-1)^n \left[ \frac{(2ph + K_f \phi^2)^{1/2} h}{Bn} K_1((2ph + K_f \phi^2)^{1/2} Bn) \right. \right. \\ & \left. \left. + \frac{((2p+2)h + K_f \phi^2)^{1/2}}{Bn} h K_1(((2p+2)h + K_f \phi^2)^{1/2} Bn) \right] \right\} \end{aligned} \quad (33)$$

Again, a number of terms from Eqs. (31) and (33) are cancelled being summed up, as in the bosonic sector.

These two expressions and the boson contributions obtained in Sect. 2 will be used in numerical investigations of symmetry behaviour.

## 4 Contribution of ring diagrams

It was shown by Carrington [9] that at  $T \neq 0$  the consistent calculation of the EP based on generalized propagators, which include the polarization operator insertions, requires that ring diagrams have to be added simultaneously with the one-loop part. These diagrams essentially affect the phase transition at high temperature and zero field [27], [9], [10]. Their importance at  $T$  and  $H \neq 0$  was also pointed out in literature [15], [16] but, as far as we know, this part of the EP has not been calculated, yet.

As it is known [17], [27], the sum of ring diagrams describes a dominant contribution of large distances. It non-negligibly differs from zero only in the case when massless states appear in a system. So, this type of diagrams has to be calculated when a symmetry restoration is investigated. To find the correction  $V_{ring}(H, T)$  at high temperature and magnetic field the polarization operators of the Higgs particle, photon and  $Z$ -boson at the considered background have to be computed. Just these calculations have been announced in Refs. [15], [16]. Then,  $V_{ring}(H, T)$  is given by series depicted in figures 1, 2. Here, a dashed line describes the Higgs particles, the wavy lines represent photons and  $Z$ -bosons, the blobs represent the polarization operators in the limit of zero momenta. As it is also known [9], in order to calculate the contribution of ring diagrams not the

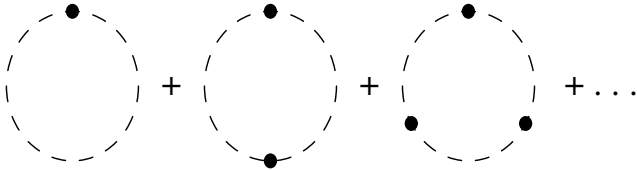


Figure 1: The Higgs field ring diagrams giving contribution to the effective potential. Blobs stand for the neutral scalar field polarization operator calculated at zero momentum.

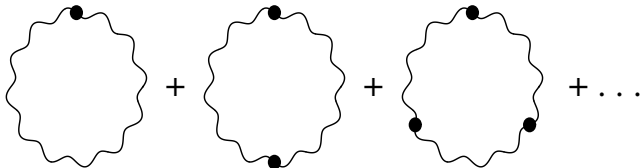


Figure 2: The photon and Z-boson ring diagrams giving contribution to the effective potential. Blobs stand for the polarization operators of the fields calculated at zero momenta.

total polarization operators  $\Pi_{\mu\nu}(k, T, H)$  but only their limits at zero momenta,  $\Pi_{00}(k = 0, T, H)$ , are sufficient. This limit, called the Debye mass, can be calculated from the EP of the special type. This fact considerably simplifies our task.

Now, let us turn to calculations of  $V_{ring}(H, T)$ . It is given by the standard expression [27], [9], [15]:

$$V_{ring} = -\frac{1}{12\pi\beta} \text{Tr}\{[M^2(\phi) + \Pi_{00}(0)]^{3/2} - M^3(\phi)\}, \quad (34)$$

where trace means the summation over all the contributing states,  $M(\phi)$  is the tree mass of the corresponding state. The functions  $\Pi_{00}(0)$  are:  $\Pi_{00}(0) = \Pi(k = 0, T, H)$  for the Higgs particle;  $\Pi_{00}(0) = \Pi_{00}(k = 0, T, H)$  - the zero-zero components of the polarization functions of gauge fields in the magnetic field taken at zero momenta. The above contributions are of order  $\sim g^3(\lambda^{3/2})$  in the coupling constants whereas the two-loop terms are to be of order  $\sim g^4(\lambda^2)$ . For  $\Pi_{00}(0)$  the high temperature limits of polarization functions have to be substituted which have order  $\sim T^2$  for leading terms and  $\sim g\phi_c T, (gH)^{1/2}T(\phi_c/T \ll 1, (gH)^{1/2}/T \ll 1)$  for subleading ones.

For the next step of calculation, we remind that the effective potential is the generating functional of the one-particle irreducible Green functions at zero external momenta. So, to have  $\Pi(0)$  we can just calculate the second derivative with respect to  $\phi$  of the potential  $V^{(1)}(H, T, \phi)$  in the limit of high temperature,  $T \gg \phi, T \gg (eH)^{1/2}$ , and then set  $\phi = 0$ . This limit can be obtained by

means of the Mellin transformation technique (see, for instance, [38]) and the result looks as follows:

$$\begin{aligned}
V^{(1)}(H, \phi, T)|_{T \rightarrow \infty} &= \left[ \left( \frac{C_f}{6} \phi_c^2 + \frac{\alpha\pi}{2\cos^2\theta_w} \phi_c^2 + \frac{g^2}{16} \phi_c^2 \right) T^2 \right] \\
&+ \left[ \frac{\alpha\pi}{6} (3\lambda\phi_c^2 - \delta^2(0)) T^2 - \frac{\alpha}{\cos^3\theta} \phi^3 T - \frac{\alpha}{3} \left( \frac{3\lambda\phi_c^2 - \delta^2(0)}{2} \right)^{3/2} T \right] \\
&- \frac{1}{2\pi} \left( \frac{1}{4} \phi_c^2 + gH \right)^{3/2} T + \frac{1}{4\pi} eHT \left( \frac{1}{4} \phi_c^2 + eH \right)^{1/2} \\
&+ \frac{1}{2\pi} eHT \left( \frac{1}{4} \phi_c^2 - eH \right)^{1/2}.
\end{aligned} \tag{35}$$

The parameter  $C_f = \sum_{i=1}^3 G_{il}^2 + 3 \sum_{i=1}^3 G_{iq}^2$  determines the fermion contribution of order  $\sim T^2$  having relevance to our problem. We also have omitted  $\sim T^4$  contributions to the EP. The terms of the type  $\sim \log[T/f(\phi, H)]$  cancel the logarithmic terms in the temperature independent parts (15), (30). Considering the high temperature limit we restrict ourselves to linear and quadratic in  $T$  terms, only.

The one else important expression, which also should be taken into account, is the linear in  $H$  term of the zero temperature EP Eq. (31), which looks as follows:

$$V_{f,l}^{(1)}(H, \phi_c)/H_0^2 = -\frac{\alpha}{2\pi} \phi^2 \sum_f K_f |q_f H|. \tag{36}$$

It significantly influences symmetry behaviour and contributes to the Debye mass in strong fields.

Now, differentiating these expressions twice with respect to  $\phi$  and setting  $\phi = 0$ , we obtain

$$\begin{aligned}
\Pi_\phi(0) &= \frac{\partial^2 V^{(1)}(\phi, H, T)}{\partial \phi^2} \Big|_{\phi=0} \\
&= \frac{1}{24\beta^2} \left( 6\lambda + \frac{6e^2}{\sin^2 2\theta_w} + \frac{3e^2}{\sin^2 \theta_w} \right) \\
&+ \frac{2\alpha}{\pi} \sum_f \left[ \frac{\pi^2 K_f}{3\beta^2} - |q_f H| K_f \right] \\
&+ \frac{(eH)^{1/2}}{8\pi \sin^2 \theta_w \beta} e^2 (3\sqrt{2}\zeta(-\frac{1}{2}, \frac{1}{2}) - 1).
\end{aligned} \tag{37}$$

The terms  $\sim T^2$  in Eq. (37) give standard contributions to temperature mass squared coming from the boson and the fermion sectors. The  $H$ -dependent term is negative since the difference in the brackets is  $3\sqrt{2}\zeta(-\frac{1}{2}, \frac{1}{2}) - 1 \simeq -0.39$ . Formally, this may result in the negativeness of the  $\Pi(0)_\phi$  for very strong fields  $(eH)^{1/2} > T$ . But this happens in the range of parameters where asymptotic

expansion is not applicable. Substituting expression (37) into Eq. (34) we obtain (in the dimensionless variables)

$$\mathcal{V}_{ring}^{\phi} = -\frac{\alpha}{3B} \left\{ \left( \frac{3\phi^2 - 1}{2} K + \Pi_{\phi}(0) \right) \right\}^{3/2} + \frac{\alpha}{3B} K \left( \frac{3\phi^2 - 1}{2} \right)^{3/2}. \quad (38)$$

As one can see, the last term of this expression cancels the fourth term in the Eq. (35), which becomes imaginary at  $3\phi^2 < 1$ . This is the important cancellation preventing the infrared instability at high temperature.

Before we proceed, let us note that Eq. (35) contains the other term (the last one) which becomes imaginary for strong magnetic fields or small  $\phi^2$ . It reflects the known instability in the  $W$ -boson spectrum which is discussed for many years in literature (see papers [12], [15], [16], [38] and references therein). But it also will be cancelled out when the contribution of ring diagrams with the unstable mode is added.

To find the  $H$ -dependent Debye masses of photons and  $Z$ -bosons the following procedure will be used. We calculate the one-loop EP of the  $W$ -bosons and fermions in a magnetic field and some "chemical potential",  $\mu$ , which plays the role of an auxiliary parameter. Then, by differentiating them twice with respect to  $\mu$  and setting  $\mu = 0$  the mass squared  $m_D^2$  will be obtained. Let us first demonstrate that in more detail for the case of fermion contributions where the result is known.

The temperature dependent part of the one-loop EP of constant magnetic field at a non-zero chemical potential induced by an electron-positron vacuum polarization is [33]:

$$V_{ferm.}^{(1)} = \frac{1}{4\pi^2} \sum_{l=1}^{\infty} (-1)^{l+1} \int_0^{\infty} \frac{ds}{s^3} \exp\left(\frac{-\beta^2 l^2}{4s} - m^2 s\right) (eHs) \coth(eHs) \cosh(\beta l \mu), \quad (39)$$

where  $m$  is the electron mass,  $e = g \sin \theta$  is the electric charge and the proper-time representation is used. Its second derivative with respect to  $\mu$  taken at  $\mu = 0$  can be written in the form,

$$\frac{\partial^2 V_{ferm.}^{(1)}}{\partial \mu^2} = \frac{eH}{\pi^2} \beta^2 \frac{\partial}{\beta^2} \sum_{l=1}^{\infty} (-1)^{l+1} \int_0^{\infty} \frac{ds}{s} \exp(-m^2 s - \beta^2 l^2 / 4s) \coth(eHs). \quad (40)$$

Expanding  $\coth(eHs)$  in series and integrating over  $s$  in accordance with formula (21) we obtain in the limit of  $T \gg m, T \gg (eH)^{1/2}$ :

$$\sum_{l=1}^{\infty} (-1)^{l+1} \left[ \frac{8m}{\beta l} K_1(m\beta l) + \frac{2}{3} \frac{(eH)^2 l \beta}{m} K_1(m\beta l) + \dots \right]. \quad (41)$$

The series in  $l$  can easily be calculated by means of the Mellin transformation (see Refs.[38], [16]). To have the Debye mass squared it is necessary to differentiate

Eq. (40) with respect to  $\beta^2$  and to take into account the relation of the parameter  $\mu$  with the zero component of the electromagnetic potential:  $\mu \rightarrow ieA_0$  [15]. In this way we obtain finally,

$$m_D^2 = g^2 \sin^2 \theta \left[ \frac{T^2}{3} - \frac{1}{2\pi^2} m^2 + O((m\beta)^2, (eH\beta^2)) \right]. \quad (42)$$

This is the well known result calculated from the photon polarization operator [29]. As one can see, the dependence on  $H$  appears in the order  $\sim T^{-2}$ . To find the total fermion contribution to  $m_D^2$  one should sum up the expression (42) over all fermions and substitute their electric charges.

To find  $m_D^2$  for  $Z$ -bosons it is sufficient to allow for the fermion coupling to the  $Z$ -field. It can be done by substituting  $\mu \rightarrow i(g/2\cos\theta + g\sin^2\theta)$  and the result differs from Eq. (42) by the coefficient at the brackets in the right-hand side which has to be replaced,  $g^2 \sin^2 \theta \rightarrow g^2 (\frac{1}{4\cos^2 \theta} + \tan^2 \theta)$ . One also should add the terms coming due to the neutral currents and the part of fermion- $Z$ -boson interaction which is not reproduced by the above substitution:

$$m_D'^2 = \frac{g^2}{8\cos^2 \theta} (1 + 4\sin^4 \theta) T^2. \quad (43)$$

Now, let us apply this procedure to the case of the  $W$ -boson contribution. The corresponding EP (temperature dependent part) calculated at non-zero  $T, \mu$  in the unitary gauge looks as follows,

$$V_w^{(1)} = -\frac{eH}{8\pi^2} \sum_{l=1}^{\infty} \int_0^{\infty} \frac{ds}{s^2} \exp(-m^2 s - l^2 \beta^2 / 4s) \quad (44)$$

$$\left[ \frac{3}{\sinh(eHs)} + 4\sinh(eHs) \right] \cosh(\beta l \mu).$$

All the notations are obvious. The first term in the squared brackets gives the triple contribution of the charged scalar field and the second one is due to the interaction with a  $W$ -boson magnetic moment. Again, after differentiation twice with respect to  $\mu$  and setting  $\mu = 0$  it can be written as

$$\frac{\partial^2 V_w^{(1)}}{\partial \mu^2} \Big|_{\mu=0} = \frac{eH}{2\pi^2} \beta^2 \frac{\partial}{\partial \beta^2} \sum_{l=1}^{\infty} \int_0^{\infty} \frac{ds}{s} \exp\left(-\frac{m^2 s}{eH} - \frac{l^2 \beta^2 eH}{4s}\right) \left[ \frac{3}{\sinh(s)} + 4\sinh(s) \right]. \quad (45)$$

Expanding  $\sinh^{-1} s$  in a series over Bernoulli's polynomials,

$$\frac{1}{\sinh s} = \frac{e^{-s}}{s} \sum_{k=0}^{\infty} \frac{B_k}{k!} (-2s)^k, \quad (46)$$

and carrying out all the calculations described above, we obtain for the  $W$ - boson contribution to  $m_D^2$  of the electromagnetic field,

$$m_D^2 = 3g^2 \sin^2 \theta \left[ \frac{1}{3} T^2 - \frac{1}{2\pi} T(m^2 + g \sin \theta H)^{1/2} - \frac{1}{8\pi^2} (g \sin \theta H) \right. \\ \left. + O(m^2/T^2, (g \sin \theta H/T^2)^2) \right]. \quad (47)$$

Hence it follows that spin does not affect the Debye mass in leading order. Other interesting feature is that the next-to-leading terms are negative.

The contribution of the  $W$ -boson sector to the  $Z$ -boson mass  $m_D^2$  is given by the expression (47) with the replacement  $g^2 \sin^2 \theta \rightarrow g^2 \cos^2 \theta$ .

Substituting the expressions (42) and (47) into Eq. (34), we obtain the photon part  $V_{ring}^\gamma$ , where it is necessary to express masses in terms of the vacuum value of the scalar condensate  $\phi_c$ . In the same way the ring diagrams of  $Z$ -bosons  $V_{ring}^Z$  can be calculated. The only difference is the mass term of  $Z$ - field and the additional term in the Debye mass due to the neutral current  $\sim \bar{\nu} \gamma_\mu \nu Z_\mu$ . These three fields -  $\phi, \gamma, Z$ , - which become massless in the restored phase, contribute into  $V_{ring}(H, T)$  in the presence of the magnetic field. At zero field, there are also terms due to the  $W$ -boson loops in Figs. 1, 2 . But when  $H \neq 0$  the charged particles acquire masses  $\sim eH$  and can be neglected.

In the restored phase, the  $W$ -bosons do not interact with the hypermagnetic field and therefore give no field dependent contributions.

A separate consideration should be spared to the tachyonic (unstable) mode in the  $W$ -boson spectrum:  $p_0^2 = p_3^2 + M_w^2 - eH$ . First of all, we notice that this mode is excited due to a spin interaction and it does not influence the  $G_{00}(k)$  component of the  $W$ -boson propagator. Secondly, in the fields  $eH \sim M_w^2$  the mode becomes a long range state. Therefore, it should be included in  $V_{ring}(H, T)$  side by side with the other considered neutral fields. But in this case it is impossible to take advantage of formula (34) and one has to return to the initial EP containing the generalized propagators.

For our purpose it will be convenient to make use of the generalized EP written as the sum over the modes in the external magnetic field [15], [16]:

$$V_{gen}^{(1)} = \frac{eH}{2\pi\beta} \sum_{l=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \sum_{n=0, \sigma=0, \pm 1}^{\infty} \log[\beta^2(\omega_l^2 + \epsilon_{n, \sigma, p_3}^2 + \Pi(T, H))], \quad (48)$$

where  $\omega_l = \frac{2\pi l}{\beta}$ ,  $\epsilon_n^2 = p_3^2 + M_w^2 + (2n + 1 - 2\sigma)eH$  and  $\Pi(H, T)$  is the radiation mass squared of  $W$ -bosons in a magnetic field at finite temperature. Denoting as  $D_0^{-1}(p_3, H, T)$  the sum  $\omega_l^2 + \epsilon^2$ , one can rewrite eq. (48) as follows:

$$V_{gen}^{(1)} = \frac{eH}{2\pi\beta} \sum_{l=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \sum_{n, \sigma} \log[\beta^2 D_0^{-1}(p_3, H, T)]$$

$$\begin{aligned}
& + \frac{eH}{2\pi\beta} \sum_{l=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \{ \log[1 + (\omega_l^2 + p_3^2 + M_w^2 - eH)^{-1} \Pi(H, T)] \\
& + \sum_{n \neq 0, \sigma \neq +1} \log[1 + D_0(\epsilon_n^2, H, T) \Pi(H, T)] \}. \tag{49}
\end{aligned}$$

Here, the first term is just the one-loop contribution of  $W$ -bosons, the second one gives the sum of ring diagrams of the unstable mode (as it can easily be verified by expanding the logarithm into a series). The last term describes the sum of the short range modes in the magnetic field and should be omitted.

Thus, to get  $V_{ring}^{unstable}$  one has to compute the second term in Eq. (49). In the high temperature limit we obtain:

$$V_{ring}^{unstable} = \frac{eH}{2\pi\beta} \{ (M_w^2 - eH + \Pi(H, T))^{1/2} - (M_w^2 - eH)^{1/2} \}. \tag{50}$$

By summing up the one-loop EP and all the terms  $V_{ring}$ , we arrive at the total consistent in leading order EP.

Let us mention the most important features of the above expression. It is seen that the last term in Eq. (50) exactly cancels the "dangerous" term in Eq. (35). So, the EP is real and no instabilities appear at sufficiently high temperatures when  $\Pi(H, T) > M_w^2 - eH$ . To make a quantitative estimate of the range of validity of the total EP it is necessary to calculate the  $W$ -boson mass operator in a magnetic field at finite temperature and hence to find  $\Pi(H, T)$ . This is a separate and enough cogent problem which is considered in detail in a separate publication. Here, we only adduce the result of  $\Pi(H, T)$  calculations [34]:

$$\begin{aligned}
\Pi_{unstable}(H, T) &= \langle n = 0, \sigma = +1 | \Pi_{\mu\nu}^{charged} | n = 0, \sigma = +1 \rangle \\
&= \alpha [26, 96(eH)^{1/2}T + i4(eH)^{1/2}T], \tag{51}
\end{aligned}$$

where the average value of the mass operator in the ground state of the  $W$ -boson spectrum  $| n = 0, \sigma = +1 \rangle$  was taken. This expression has been computed in the limit  $eH/T^2 \ll 1, B = M_w(H, T)/T \ll 1$ , which is a good approximation since, as it will be shown below, typical inverse temperatures for the symmetry restoration are  $B \sim 0.1 - 0.3$ . Side by side with the real part responsible for the radiation mass squared the expression (51) contains the imaginary one describing the decay of the state. Its value is small as compare to the real part and of order of the usual damping constant at high temperature. So,  $\text{Im } \Pi(H, T)$  can be ignored in our problem. The radiation mass squared is positive and acts to stabilize the spectrum. At  $H = 0$  no screening is produced in one-loop order, as it should be at finite temperatures for transversal modes [17]. Thus, we see that at high temperatures the effective  $W$ -boson mass squared,  $(M_w^2)_{eff} = M_w^2 - eH + \Pi(H, T)$ , is positive. Therefore, no conditions for  $W$ -boson condensation discussed in Refs. [23], [25] are realized. With this result obtained we conclude that our EP is real for temperatures corresponding to the phase transition epoch.\*<sup>1</sup>

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<sup>1</sup> Expression (51) disagrees with the corresponding one of Ref. [30] where the average value

## 5 Effective potential of the restored phase

Having obtained the EP at  $\phi \neq 0$  we are able to investigate the form of its curve in the broken phase and determine the type of the EW phase transition for different  $m_H, h$ . To describe more precise the restored phase one has also to calculate radiation corrections to the external hypermagnetic field  $H_Y$  at high temperature. Before doing that let us remind that at  $\phi = 0$  the field  $H_Y$  is completely unscreened. This means that in a covariant derivative describing interaction with the external field one should include the  $U(1)_Y$  term:  $D_\mu = \partial_\mu + \frac{1}{2}g'B_\mu^{ext}$ . We set the potential as before,  $B_\mu^{ext} = (0, 0, H_Y, 0)$ .

In the restored phase  $W$ -bosons do not interact with  $H_Y$ . The field dependent part of the EP  $V(\phi = 0, H_Y, T)$  is non-zero due to the contributions of fermions and scalars. However, the fermion part depends logarithmically on temperature ( $\sim \frac{(g'/2)^2}{4\pi} H_Y^2 \log T/T_0$ ) and can be neglected as compared to the tree level term  $\frac{1}{2}H_Y^2$ . The scalar field contribution to the one-loop EP is

$$V_{sc}^{(1)}(H_Y, T) = - \frac{(g'/2)^2 H_Y^2}{24\pi^2} \ln(T/T_0) + \frac{((g'/2)H_Y)^{3/2} T}{6\pi} + O(1/T). \quad (52)$$

The term logarithmically dependent on  $T$  can again be neglected but the linear in  $T$  part must be retained. Since “hyperphotons” are massless in the restored phase we also include the contribution of the corresponding ring diagrams:

$$V_{restored}^{ring}(H_Y, T) = -\frac{T}{12\pi} \left[ \frac{2}{3} (g'/2)^2 T^2 + m_{D_f}^2 - \frac{((g'/2)H_Y)^{1/2} T}{2\pi} - \frac{1}{8\pi^2} (g'/2)H_Y \right]^{3/2}, \quad (53)$$

where  $m_{D_f}^2 = \frac{1}{24}g'^2 T^2 \sum_{f(R,L)} Y_f^2$  is the sum over the fermion contributions to the Debye mass of the “hyperphotons”,  $Y_f$  are the hypercharges of  $R$ - and  $L$ - leptons and quarks. Both these expressions have been calculated in a way described in the previous sections.

For convenience of numerical investigations let us rewrite Eqs. (52) and (53) in terms of the dimensionless variables  $h, B$ :  $V(H_Y, T)_{restored} = (H_0)^2 v_{restored}(h, B)$ ,

$$v_{restored}(h, B) = \frac{1}{2} \frac{h^2}{\cos^2 \theta} + \frac{\alpha}{3\sqrt{2}\cos^3 \theta} \frac{h^{3/2}}{B} - \frac{1}{3} \frac{\alpha}{B} \left[ \frac{7}{6} \frac{4\pi\alpha}{\cos^2 \theta B^2} - \frac{h^{1/2}}{2\sqrt{2}\pi B \cos \theta} - \frac{h}{16\pi^2 \cos^2 \theta} \right]^{3/2}, \quad (54)$$

of the gluon polarization operator in an abelian chromomagnetic field was calculated in a weak field approximation and  $\Pi(H, T)$  has been found to be zero. Most probably, the discrepancy is the consequence of the calculation procedure adopted by these authors when the gluon polarization operator was computed at zero external field and then its average value has been calculated in the state  $|n=0, \sigma=+1\rangle$ . Our expression is the high temperature limit of the mass operator which takes account of the external field exactly.



where  $\alpha = e^2/4\pi$  and  $h_Y = h/\cos\theta$ .

## 6 Symmetry behaviour in a hypermagnetic field

Now, let us investigate the EW phase transition in the hypermagnetic field for different values of  $m_H$ . It can be done by considering the Gibbs free energy in the broken,  $G_{broken}(H^{ext}, \phi_c, T)$ , and the restored,  $G_{restored}(H_Y^{ext}, T)$ , phases [4], [5]. The first order phase transition can be determined from two equations:

$$G_{restored}(H_Y^{ext}, T, 0) = G_{broken}(H^{ext}, T, \phi(H^{ext})_c), \quad (55)$$

describing the advantage of the broken phase creation, where  $\phi(H)_c$  is a scalar field vacuum expectation value at given  $H, T$  which has to be found as the minimum position of the total EP,

$$\frac{\partial V(H, T, \phi_c)^{total}}{\partial \phi_c} = 0. \quad (56)$$

Hence the critical field strength can be calculated. In this expression (and below) we write for brevity  $H$  instead  $H^{ext}$ .

Having obtained the EP in the restored phase, the one-loop EP described by formulae (16), (22), (28)-(33) and the ring diagram contributions  $V_{ring}$  we are going to investigate the symmetry behaviour. First, we consider the total EP as the function of  $\phi^2$  at various fixed  $H, T, K$  and determine the form of the EP curves in the broken phase. In this way it will be possible to select the range of the parameters when the first order phase transition is realized. After that the temperature  $T_c$  at given field strength  $(H_Y)_c$  will be estimated using Eqs. (55), (56).

As usually [35], to investigate symmetry behaviour we consider the difference  $\mathcal{V}' = Re[\mathcal{V}(h, \phi, K, B) - \mathcal{V}(h, \phi = 0, B)]$  which gives information about symmetry restoration.

In what follows it will be also convenient to express the conditions of the phase transition in terms of the dimensionless variables  $h, B, \phi$ , taking into account the relation  $h_Y = h/\cos\theta$ . Then, the Gibbs free energy

$$G_{broken}(h^{ext}, \phi, B) = \frac{h^2}{2} + v'(h, \phi, B) - hh^{ext}, \quad (57)$$

has to be expressed through  $h^{ext}$  by using the equation

$$h^{ext} = h + \frac{\partial v'(h, \phi, B)}{\partial h}, \quad (58)$$

where  $v'$  describes the one-loop and the ring diagram contributions to the EP. The phase transition happens when the relation holds,

$$\frac{h^2}{2} \tan^2 \theta = v'_{restored}(h, B_c) - v'_{broken}(h, \phi_c, B_c). \quad (59)$$

The function  $v'_{restored}$  is given by Eq. (54). We also have substituted the field  $h^{ext}$  by  $h$ .

The results on the phase transition determined by numerical investigation of the total EP are summarized in Table 1.

h	$K$	$T_c(GeV)$	$\phi_c(h, T_c)$	$\phi_c^2(h, T_c)$	$R$	$M_w(h, B_c)$
0.01	0.85	106.47	0.301662	0.091	0.69699	0.327235
0.01	1.25	122.21	0.181659	0.033	0.36567	0.230086
0.01	2	145.56	0.094868	0.009	0.16033	0.186168
0.1	0.85	108.58	0.275681	0.076	0.62459	0.245186
0.1	1.25	123.54	0.130384	0.017	0.25963	0.112721
0.1	2	148.39	0.031623	0.001	0.05242	0.126315
0.5	0.85	108.89	0.248998	0.062	0.56253	0.49938 i
0.5	1.25	second	order	phase	transition	
0.5	2	second	order	phase	transition	

Table 1.

In the first column we show the hypermagnetic field strength in the broken phase (in dimensionless units). In the second and third the mass parameter  $K = m_H^2/M_w^2$  and the critical temperature of the first order phase transition are adduced. Next two columns give the local minimum positions  $\phi_c(H, T_c)$  and their squared values at the transition temperatures. The last two columns fix the ratio  $R = 246 \text{ GeV } \phi_c(h, T_c)/T_c$ , determining the advantage of baryogenesis, and the  $W$ -boson effective mass calculated in the local minimum of the EP at the corresponding field strengths and the transition temperatures.

As it is seen, the increase in  $h$  makes the phase transition weaker (not stronger as it was expected in Refs. [4], [5] by analogy to superconductivity in the external magnetic field). The ratio  $R$  is less than unit for all the field strengths, whereas the baryogenesis condition is  $R > 1.2 - 1.5$  [19]. Thus, we come to the conclusion that external hypermagnetic fields do not make the EW phase transition strong enough to produce baryogenesis. Moreover, for strong fields the phase transition is of second order for all the values of  $K$  considered. These are the main observations of our numerical investigations.

Let us continue the analysis of data in the Table 1. For the field strengths  $H > 0.1 - 0.5 H_0 (H_0 = M_w^2/e)$  the phase transition is of second or weak first-order. The  $W$ -boson effective mass squared (in dimensionless units)  $M_w^2(\phi_c, h, B_c) = \phi_c^2(h, B_c) - h + \Pi(h, B_c)$  is positive for  $h = 0.01$  and  $h = 0.1$ . Therefore, the local minimum is the stable state at the first order phase transition. For stronger fields, when the second order phase transition happens, the effective  $W$ -boson mass becomes imaginary. This reflects the known instability in the external magnetic field which exhibits itself even when the radiation mass of the tachyonic mode is included. But it does not matter for the problem of searching for the strong

first order phase transition in the external hypermagnetic field investigated in the present paper. The instability has to result in the condensation of  $W$ - and  $Z$ -boson fields at high temperature.

In Refs. [4], [6], [5], [7] it has been determined that the strong hypermagnetic field increases the strength of the first order phase transition and in this case baryogenesis survives in the SM. Our results are in obvious contradiction with this conclusion. To explain the origin of the discrepancies let us first consider Refs. [4], [5] where a perturbative method of computations has been applied. These authors, considering the EW phase transition, have allowed for the influence of the external field at tree level, only. That corresponds to the usual case of superconductors in the external magnetic field, and, as a consequence, they observed the strong first order phase transition. In fact, the type of the phase transition was just assumed, since no investigations of the EP curve with all the particles included for different  $H_Y, T$  have been carried out. In the former paper, the qualitative estimate of the phenomenon considered was given, whereas in the latter one the quantitative analysis in one-loop approximation for the temperature dependent part of the EP has been done. Actually, in both these investigations the influence of the external field was reduced to consideration of the condition (59) fixing the transition temperature. The role of fermions and  $W$ -bosons in the field was not investigated at all. However, as we have observed, the fermions (heavy and light) are of paramount importance in the phase transition dynamics. Just due to them the EW phase transition becomes of second order in strong fields (for the values of  $K$  when it is of first order in weak fields).

In Refs. [6], [7] the phase transition was investigated by the method combining the perturbation theory and the lattice simulations. As the first step in this approach the static modes are maintained in the high temperature Lagrangian. The fermions are nonstatic modes and decoupled. So, no reflections of the fermion properties in the external fields and, hence, no information on the EP curve could be derived in this way. The only fermion remainder is the  $t$ -quark mass entering the effective universal theory [19], [6], [7]. In our analysis, it has been observed that not only heavy but also light fermions are important in strong external fields at high temperature. In fact, for various field strengths the fermions with different masses are dominant and we have taken account of all of them. Moreover, we have allowed for all the ring correlation corrections in the external field that also influences symmetry behaviour.

We would like to notice that our perturbative results for the values of  $K \sim 0.8 - 0.9$  are reliable. They are in agreement with nonperturbative analysis at zero field. The external field is taken into account exactly. For these masses of the Higgs particles we observed the change of the first order phase transition to the second order one with increase in the field strength. The same behaviour takes place for  $K > 1$  when perturbative analysis may be not trusty. But, as we have discovered, the general picture of the field effects is only quantitatively changed for heavy scalar particles. In this case also the first order phase transition in

weak fields becomes of second order one for strong fields. These circumstances convince us that the assumption of Ref. [4] that the hypermagnetic fields is able to make the weak first-order EW phase transition strong enough is not approved by the detailed calculations.

## 7 Discussion

In Refs. [4], [5] the influence of strong external hypermagnetic field on the EW phase transition has been taken into account in tree approximation. Further studying of the phenomenon, naturally, has to allow for the radiation and correlation corrections. This is the problem that we have addressed to in the present paper. The main idea was to determine the form of the EP curve in the broken phase and find the range of the parameters  $H_Y, K$  when the EW phase transition is of first order. To elaborate that the consistent EP including the one-loop and ring diagrams of all the fundamental particles has been constructed. As we have seen, the role of fermions and ring diagrams in the external field is crucial when the structure of the broken phase is described. The external field was taken into consideration exactly through Green's functions. The minimum of the EP was found to be stable at sufficiently high temperatures when the first order phase transition happens. This important property is fulfilled when the ring diagrams of the tachyonic mode are included. As a result, no conditions for  $W$ - and  $Z$ -boson condensates are realized at high temperatures at the first order phase transition. The condensates could be generated for stronger fields at the second order phase transition. But in this case baryogenesis does not survive.

The influence of strong magnetic fields on the vacuum at high temperature is a complicate corporative effect described by the total EP. At some chosen values of  $H, T, K$  the different terms of it are dominant. To better understand the role of fermions in symmetry behaviour let us adduce two terms of the asymptotic expansion of the EP in the limit of  $T \rightarrow \infty, H \rightarrow \infty$ . The first one is the term  $\sim H^2 \log \frac{T}{m_f}$ . Due to this term the light fermions are dominant at high temperature. The second term can be derived from the expansion of the zero temperature part Eq. (31). This expression side by side with the leading term  $\sim H^2 \log \frac{eH}{m_f}$ , which due to a "dimension parameter trading" is replaced by the above written term, contains the subleading one  $\sim -eHm_f^2 \log \frac{eH}{m_f^2}$  (for details see Ref. [32]). This term acts to make "heavier" the Higgs particles in the field. As a result, the second order temperature phase transition is stimulated due to strong fields. As it is well known at zero field, the correlation corrections relax the strength of the first order phase transition [9] - [11]. This property is provided by the structure of the  $V_{ring}$  term of the EP (34), independently of the field presence. The field-dependent terms of the Debye masses of the scalar (37), photon (47) and  $Z$  particles are negative that decreases the mass values. Therefore, the field

acts to make weaker the effect of correlations as the zero field case is compared. But nevertheless the relaxation effect as such holds. We also have investigated the influence of different parts of the EP on symmetry behaviour. It was discovered that the change of the phase transition kind with increase in  $H$  is due to the fermion temperature part of the EP. These remarks help us to have a notion about the role of fermions and correlations in strong fields.

In papers [6], [7] the EW phase transition in the hypermagnetic field has been investigated by means of the method combining perturbation theory and lattice simulations and the main conclusions of Refs. [4], [5] were supported. In the former two papers, because of peculiarities of the calculation procedure adopted, the effects of the external field due to fermions as well as the correlation corrections have not been allowed for. So, from the point of view of the present analysis these results also do not reproduce correctly the behaviour of the EP curve in the broken phase.

The values of the Higgs boson mass investigated in the present paper correspond to the cases when perturbative results are reliable ( $K = 0.85$ ) and may be not trusty ( $K = 1.25, 2$ ). However, since the external field is taken into account exactly its effects do not depend on the specific  $K$  values. As we have seen, an increase in  $H_Y$  makes the EW phase transition of second order for the field strengths  $H_Y \sim 0.5 \cdot 10^{24} G$  for all the mass values investigated. For weaker fields the phase transition is of first order but the ratio  $R = \phi_c(H, T_c)/T_c$  is less than unit. Hence we conclude that baryogenesis does not survive in the minimal Standard Model in the smooth external hypermagnetic fields.

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